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Research on theoretical ideas (well-posedness, feedback control, stability of inverse problems, form of damping in structures) and associated computational methods was performed. Topics include (i) fluid/structure interaction and flow control with applications to Navier-Stokes steady flow, Burgers equation, advection-diffusion, porous media flow, and acoustics/noise suppression (ii) inverse problems for detection of semiconductor defects, electrical impedance tomography, damping and ''smart'' material actuators in composite structures (iii) linear, nonlinear and homogenization models for beams, plates, shells and PZT actuators and sensors.

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Report prepared by:
H. T. Banks

Brief Summary of Research

Research was pursued on theoretical, computational and/or experimental aspects of the following topics:

I. Fluid / Structure Interaction / Flow Control

a) Navier-Stokes – Steady Flow

i) Driven cavity

ii) Channel with abrupt expansion

iii) High pressure vapor transport reactors (manufacturing of microelectronic devices)

b) Burger's Law

c) Advection – Diffusion

d) 3-D Porous Media Flow (groundwater flow, domain decomposition, parallel methods)

e) Acoustics / Noise Suppression

II. Inverse Problems (Identification, Parameter Estimation)

a) Laser Beam Induced Current Techniques (detection of Semiconductor Defects)

b) Computational Techniques – Regularization, Spectral Methods including Tau-Legendre, Costate Methods, Multigrid, I.D. in Frequency Domain for DPS

c) Electrical Impedance Tomography

d) Materials and Structures – Damping, "Smart" Material Components / Sensors / Actuators

III. Structures

a) Models for Beams, Plates, Shells, Multicomponent Structures, Linear, Nonlinear, Homogeneation for Truss Structures

b) Coupled Torsion and Bending in Vibrations

c) Damping and Its Role in Feedback Control

d) "Smart" or Adaptive Materials: Modeling of PZT Actuators / Sensors (Self Sensing Actuators; Collocated Sensing / Actuation)

e) Fluid / Structure Interaction (Active Control for Vibration Suppression)

IV. Theoretical Methods

a) Approximation Methods for Linear / Nonlinear Feedback Control (LQR) Problems with Unbounded Input / Output Operators

b) Methods for Estimation of Spatially / Time Varying Parameters in DPS

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K. Ito
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Research Summary

I. Modeling and Approximation for Acoustic/Structure Control (Banks, Fang, Fakhroo, Ito, Y. Wang)

In joint efforts with R.J. Silcox (NASA Langley) and R.C. Smith (ICASE), several members of the AFOSR supported research team have studied modeling [BPS], approximation methods and active noise control for an acoustic/structure model that describes the interaction between an interior noise field and the motion of a beam, plate or thin shell boundary controlled by piezoceramic actuators. The purpose of this study was to provide a fundamental understanding for such a system so that an active noise control methodology can be developed. The project was motivated by the need for reduction of noise produced by fuel efficient turboprop engines to provide suitable environment for passengers, and experiments in support of this effort are being designed and carried out at NASA Langley Research Center. The model consists of the wave equation in the cavity, a beam, plate or cylindrical shell equation on part of the boundary and an interaction mechanism between the two. The interaction is described by continuity of velocity and the back pressure on the structural boundary by the noise field. The structure is controlled by the attached piezoceramic patches that generate pure bending moments on the beam or plate. Modeling aspects are presented in [BFS] for the two-dimensional acoustic cavity case. Due to the moment control through the piezoceramic patches, this control system has unbounded input. Approximation methods in the context of an LQR state space formulation are discussed and numerical results are presented in [BFSm], [BFSS], [BSS] to demonstrate the effectiveness of this approach in computing feedback controls for noise reduction. Theoretical aspects of the control system are still under investigation. In connection with controllability of the system, members of the group have studied exponential stability of this uncontrolled system with elastic boundary conditions. Although it is known that the wave equation with boundary damping mechanism and the beam equation with Kelvin-Voigt damping are each exponentially stable individually, it is not clear at all that the coupled system possesses the same property. It is shown in ([BaFI]) that this coupled system is indeed exponentially stable, and the result can be abstracted to obtain exponential stability for coupled systems whose coupling satisfies certain conditions. A study is planned to investigate the decay rates of the wave equation with various types of boundary damping mechanism.

The initial efforts with 2-D models of fluid/structure interactions controlled by piezoceramic actuators led to extensive efforts by the group on two fronts: (i) the development of careful control and approximation techniques for piezoceramic actuation in 1-D, 2-D and 3-D structures including beams, plates and thin cylindrical shells and (ii) methods for parameter estimation and feedback control in the resulting fluid/structure systems. In [BWIS] the authors demonstrated the efficacy of computational methods developed to deal with fundamental modeling issues related to mass/stiffness/damping loading of structures with piezoceramic patch actuators. The findings were coupled with extensive experimental efforts to validate the findings. The importance of actuator loading as well as mode of force/moment input to the structures led to extensive efforts for modeling of structure/actuator coupling in beams, plates and curved surfaces, including cylindrical thin shells. These investigations resulted in a number of fundamental contributions to the emerging field of control and

sensing in smart or adaptive materials technology. These findings are reported in [BSW1], [BS1], [BSW2], [BLS]. These findings were used to develop models, parameter estimation techniques, and feedback control gain computational methods for the 3-D version of the structural/acoustics interaction problem outlined above. The results are reported in [BS2], [BS3], [BS4].

An alternative approach to piezoceramic actuators on the transmitting structure in the noise reduction problem involves a *secondary* noise source as controller. In this approach one introduces a secondary noise into the interior cavity to interact in a (hopefully) destructive manner with the offending primary noise. In [FF], a periodic linear quadratic tracking problem formulation of this problem was presented. An approximation framework based on the Legendre-Tau method for approximating the control system was developed and theoretical questions such as stability and convergence of the numerical scheme and the convergence of the approximate control system to the original one were considered. In an ensuing paper [BFa], which is based on the results obtained in the thesis, some of the arguments for convergence of the scheme and convergence of the approximate control system were modified and some questions such as "adjoint convergence of the semigroup" were successfully answered.

F. Fakhroo also worked on the question of optimal location for the controllers for the same acoustic problem. For this problem, the cost function that was formulated for the control problem was first optimized over the appropriate class of control functions, and then expressed as a function of location of controls and the optimal cost function was minimized over different locations of controls. This sensitivity analysis was also pursued for other parameters that define the controls such as radius of input source (speaker) and number of controls.

II. Experimental Program (Inman, Slater, Y. Wang, Banks)

A series of experiments have been performed in support of theoretical developments in the modeling and identification of damping mechanisms in basic structural elements. In particular, an effort was made to construct an experimental test article that would eliminate damping or energy dissipation through the boundary of a clamped beam configuration. An apparatus was constructed with the intent of producing a perfectly clamped boundary condition. In particular, rather than clamping a beam to a fixed ground, a block of aluminum was machined down to a block plus a beam. In this way it was hoped that the boundary of the clamped beam would not slip at the point of cantilever, reducing a common source of frictional damping found at the interface in a standard clamped free arrangement.

Another source of external damping which commonly spoils attempts to measure internal damping is the wire leads associated with most sensors, such as an accelerometer. The difficulty was removed by using a proximity sensor which measures displacement without physically contacting the beam. Various configurations of this test article were tested (over 144 impulse response tests plus 72 free decay tests). The specific configurations tested include:

- beam with and without an "ideal clamp"
- beam with and without piezoceramic actuator/sensor

The free decay experiments of the active structure consisted of driving the beam with the surface mounted piezoceramic at its base and measuring the response as it decays once the sinusoidal excitation is turned off.

A series of experiments to investigate beam vibrations in thermally varying environments

for identification and control were performed. These efforts were in conjunction with scientists at AFAL (Air Force Astronautics Laboratory) at Edwards Air Force Bases and involved performing structural dynamics experiments with AFAL's thermal/vacuum chamber.

Working in conjunction with D.J. Inman and his students in the Mechanical Systems Laboratory at State University of New York at Buffalo, investigators on this grant carried out a series of experiments to study coupled torsional and bending in beams.

III. Role of Damping in Control (Banks, Inman)

It is extremely important that the internal damping of a structural element (i.e., beam, plate, etc.) be modeled as precisely as possible, when performance of a closed loop system as well, as its stability properties are important. Poorly estimated damping and/or poorly modeled damping can spoil both the performance and stability properties of a closed loop system. The same results, summarized here, illustrate that added damping, such as a constrained layer viscoelastic material can be used to enhance closed loop stability robustness as well as improve closed loop performance. The main result is reported in Banks and Inman ([BIn1]) and is summarized here only for the single degree of freedom cases with velocity feedback control for brevity and clarity. The basic idea is very straightforward. A closed loop system with velocity feedback can be written as

$$m\ddot{x} + (c+g)\dot{x} + kx = 0$$

where m, c, and k are the system mass, damping and stiffness coefficients respectively, and where g is the control gain. Suppose for a moment that c is *under estimated*. Then the value of the gain g chosen to produce a desired performance (say decay rate) will result in an actual velocity coefficient, $c+g$, smaller than expected and the desired performance will not be obtained. Next suppose the damping coefficient c is actually larger than measured and the objective of the closed loop system is a very fast speed of response (such as in manufacturing). In this situation positive feedback would be used $(-g)$ to drive the velocity coefficient $(c+g)$ as close to zero as possible without becoming negative. However if c is not as large as measured, g might well be chosen in a manner such that $c+g$ is negative, and the closed loop system would then become unstable.

This brings us to the second point, that the internal damping essentially determines the gain margin, or robustness, of the closed loop system and hence adding open loop damping (e.g., a constrained viscoelastic layer) improves stability robustness and enhances performance.

Another important point is the modeling of the form of an internal damping mechanism. This cannot be seen from the simple single degree of freedom model used above, but is fairly clear from an Euler Bernoulli beam, as large differences occur in the spectrum of beams with viscous damping versus those with strain rate damping (or for those with both). Hence it is extremely important to identify the proper form of the damping mechanism as well as the model's particular parameter values. Again the energy dissipation mechanism determines the closed loop robustness of a structure and knowing the energy dissipation depends on both the form of the model as well as the numerical values of the coefficients of the model. In the "competition" in structural dynamics between those who prefer a finite element approach versus those, like ourselves, who prefer the partial differential and integral modeling approach, it is a key point that traditional finite element models can only estimate a damping coefficient and not a model. Hence in these situations where more than one form

of internal damping is present, a finite element analysis is doomed to provide a structural dynamic model which will result in poor or unstable closed loop performance.

In summary, both the form of the models of internal damping of a structure and the coefficient associated with these forms have a profound influence on the stability and performance robustness of the closed loop response of the system. It appears that only a control system designed for a plant based on a fully distributed parameter approach to modeling structures can provide high performance closed loop systems.

IV. Self Sensing Actuation (Inman)

While not directly funded by this effort, and important "spin off" effort by the PI's contributed a significant break through in smart structure technology. In particular a self sensing actuator was developed based on piezoelectric constitutive relationships. This result allows a single piezoceramic element (any size patch of PZT) to perform simultaneous sensing and actuation. This analog function is described in detail in Dosch, Inman and Garcia [DIG]. The basic idea rests on first principles and does not involve any spatial or temporal approximation. The method uses a simple voltage source and capacitance model of a piezoceramic and is able to separate that part of the strain, or strain rate, resulting from the control voltage input (known) from that due to strain, or strain rate, resulting from motion of the structure. The method has been derived and experimentally verified. It is a useful technique for providing perfectly collocated control in those circumstances where the required control strain and the strain resulting from motion of the structure are of the same order of magnitude. The significance of collocated control lies in its inherent closed loop stability robustness for velocity feedback schemes. In particular with perfectly collocated control, no dynamics occur between the sensor and actuator. This usually renders the closed loop coefficient self adjoint resulting in the approximating finite dimensional control scheme having positive definite and symmetric coefficients. This results in an asymptotically stable closed loop system regardless of gain.

V. Flow Control (Ito, Kang)

The objective of these investigations was to explore mathematical and computational ideas for the development of control mechanisms in fluid flows. In [DI] the authors formulated optimal control problems for steady flow governed by the Navier-Stokes equations for driven cavity and flow through a channel with sudden expansion. Necessary optimality conditions are derived and an algorithm based on the augmented Lagrangian method is developed and analyzed. The algorithm was successfully tested for optimal control of recirculation fields of both cavity and channel flows. A dissipative nonlinear feedback synthesis for regulating the motion of fluid flow to a desired equilibrium state is developed in [IK1]. Here the authors considered control systems governed by

$$\frac{d}{dt}x(t) + \epsilon Ax(t) + F(x(t)) = Bu(t) + f(t), \quad x(0) = x_0 \in X,$$

where $x(t) \in X$ denotes the state function, A is a nonnegative self-adjoint operator on a Hilbert space X representing the term due to the viscosity, F is a locally Lipschitz mapping from $V = \mathrm{dom}(A^{1/2})$ into V^* representing the convective term and $B \in \mathcal{L}(R^m, X)$ is a control input operator $Bu = \sum_{i=l}^{m} b_i u_i(t)$ with $b_i \in X$. A class of problems described by this includes the incompressible Navier-Stokes equations, Burgers equations and advection-diffusion equations. For the Navier-Stokes equation $x(t)$ stands for the velocity field. The

proposed control law is based on the co-located rate sensors form

$$\gamma(t)B^*x(t) = \gamma_i(t)(b_i, x(t))_X.$$

In practice, the control distribution functions b_i are locally supported and thus the control signals $(b_i, x(t))_X$ involve local measurements of the state $x(t)$. This form of feedback solution is optimal for the linearized control system at the equilibrium state for properly chosen quadratic cost functionals. Then, a feedback synthesis for $\gamma(t) \geq 0$ is obtained by utilizing nonlinear dynamic programming techniques. Properties of the proposed feedback law as well as the well-posedness of the closed-loop system are established. A feasibility of the proposed control law is demonstrated for the Burgers equation and Navier-Stokes equation. In order to see how the controller effects the transient flow, asymptotic behavior, and energy dissipation of flow, several numerical computations are reported for the both Burgers equation and two dimensional Navier-Stokes equations. In [IK2] the authors developed a dissipative pseudo-spectral method for solving the two-dimensional Navier-Stokes equations with periodic boundary conditions in vorticity-stream function formulation. The proposed method preserves the zero dissipation properties and can be efficiently implemented using the fast Fourier transform. Here, by the zero dissipation properties we mean $\int_\Omega (u \cdot \nabla\omega)\omega\, dx = 0$ and $\int_\Omega (u \cdot \omega)\phi\, dx = 0$, where u is the velocity field, $\omega = curl \cdot u$ is the vorticity, and ψ is the stream function of fluid flow. The zero dissipation properties are intrinsic dynamical properties and play essential roles for the convergence analysis and long time integration. A weak variational formulation of the two-dimensional Navier-Stokes equations of vorticity-stream function form was extended to treat nonhomogeneous boundary conditions in [IK3]. This state space formulation is topologically equivalent to the velocity-pressure formulation and enables one to investigate the effects of the control law proposed in [IK1] using the pseudo-spectral method developed in [IK2].

In [BIK], the authors developed several numerical algorithms for computing feedback control laws for stabilizing the solution of the Burgers equation. The Burgers equation is a simple one-dimensional model for convection-diffusion phenomena such as shock waves, traffic flows and supersonic flow around airfoils. Within finite time, the solution of the Burgers equation produces steep gradients ("weak shock"). In order to smooth out the steep gradients as well as to obtain a desired degree of stability for the closed-loop nonlinear system, several feedback control laws were constructed based on linearization techniques and linear quadratic regulator theory. The controllers were designed for acting on a part of domain and on boundary. Numerical algorithms for computing optimal feedback gain functions are developed.

VI. Inverse Problems (Banks, Ito, Fang, Kunisch, Wade, Y. Wang)

A. Elliptic PDE. The objective of this study was to investigate the identifiabity of parameters appearing in elliptic PDEs and develop and analyze reconstruction algorithms. In [IK4] Ito and Kunisch studied the injectivity of the permeability coefficient to the solution mapping in groundwater flow modeled by second order elliptic PDEs. A variational method for the transport equation is employed to obtain the Hölder continuity of the inverse mapping under appropriate identifiability conditions. In [IK5] Tikhonov regularization and regularization due to norm constraints are analyzed for robust reconstruction algorithms. A model function technique, which is obtained by the sensitivity analysis of the least square solutions with respect to the regularization parameter, is proposed to iteratively determine

an optimal regularization parameter or norm bound. Feasibility of the proposed techniques is demonstrated for the inverse problem in one dimensional groundwater flow problems. A problem of optimal input design for the inverse problem in one dimensional groundwater flow was studied in [IK6]. Several sensitivity measures of the inverse mapping with respect to the measurements are proposed and their validity and effectiveness are analyzed. The input functions are chosen in an iterative manner so that the sensitivity measure is maximized at each iterate.

B. Modeling and analysis of laser beam induced current technique for semiconductors. Joint with S. Busenberg (Harvey Mudd College), K. Ito and W. Fang have been studying the mathematical aspects of a newly developed nondestructive optical technique for detection of defects in semiconductor materials [BFI], [FI1], [FI2]. The technique, called laser beam induced current (LBIC), was first developed by scientist J.J. Bajaj and his colleagues at the Science Center of Rockwell International, with whom cooperation has been made during the course of this research. The LBIC image consists of measurements of the total current flowing out through ohmic contacts induced by an applied laser beam. The laser beam scans the material domain and thus one obtains the LBIC image $i(x)$ as a function of position x of the applied laser beam. To formulate an inverse problem a mathematical model equation based on the drift-diffusion model for semiconductors is developed to describe the LBIC image, where the interior property of the material is represented by the impurity doping profile of the semiconductor material. This new technique has been experimentally shown to possess many advantageous features over conventional techniques, but was still lacking in quantitative description. To this end, a mathematical model for the LBIC technique is established in [BF1], which consists of the stationary drift-diffusion model for semiconductors with an extra source term representing the applied laser power. Existence result for the model is established, and uniqueness for certain simplified cases. In this model, the relation between the desired interior structure of a semiconductor material and its LBIC image is described by an inverse problem for a nonlinear elliptic system of PDE's. In [BF1], an approximate model is proposed, with a verification of its validity, for further analysis of this inverse problem. With this approximate model, a useful transformation is found so that the inverse problem can be represented as relations between spatial functions in the PDE system, and it is proved in [FI1] by using this transformation that the LBIC technique is reliable for detecting presence of defects described by nonhomogeniety in the impurity doping profile of the material. Recently, reconstruction algorithms of the doping profile from the LBIC image is developed in [FI2]. The one-dimensional algorithm, based on a complete analysis of the inverse problem, has reduced the problem of finding a function to finding two constants, hence the computation for the reconstruction becomes quite convenient. For a special two-dimensional case, an alternate direction iteration algorithm is proposed, and numerical results are presented to illustrate the effectiveness of the algorithms. For the time-dependent version of the model, W. Fang and K. Ito also study the existence and uniqueness ([FI3]) and its asymptotic behavior as time goes to infinity ([FI4]). These results also extend existing results on the drift-diffusion model for semiconductors.

In a related effort and as a continuation of his Ph.D. thesis, W. Fang has also completed the manuscript [F] on the study of contact resistivity of multilevel transistors. The resistivity between the two materials in contact can be identified from a one-point boundary voltage measurement, and it is shown that the this map is monotone and the limits of the voltage measurement as the resistivity goes to zero/infinity are characterized. These results are

useful in applications.

C. Least-squares inverse problems. A substantial part of the research activities also involved or were motivated by the problem of numerically solving least-squares type inverse problems for partial differential equations (PDE). For such problems, one important part of any numerical approach is the means by which the PDE are approximated. This issue and related convergence issues were the focus of some of the research, as discussed below. Moreover, when the unknown parameters being sought in the inverse problem are of a *distributed nature* (i.e., they are functions of one or more variables and not just a finite set of constants), then they too must be approximated and the resulting least-squares minimization problem can have an arbitrarily large number of degrees of freedom. Thus in addition to the approximation issues mentioned above, much the research was devoted to investigations of ways to make these large-scale inverse problems more computationally tractable.

i) Approximation of PDE. The "Weak Tau" method. A new class of numerical methods, the "weak Tau" method, for solving PDE was developed. These methods are motivated by a desire to combine certain advantages of the Galerkin method (for example, minimal smoothness requirements) with the sometimes dramatic performance characteristics of "spectral" methods based on Chebyshev and Legendre polynomials. A comprehensive theoretical framework for the weak Tau method for abstractly parabolic PDE, including convergence properties in the context of inverse problems, is described briefly in [BW1] and developed fully in [BW2]. In the latter work, detailed numerical comparisons of the weak Tau/Legendre method with Galerkin/spline methods is reported. Also, a more detailed comparison of this theoretical framework to that of the Galerkin method is presented in [BW3].

Some of the ideas of the weak Tau methodology for parabolic PDE are also being applied to hyperbolic PDE. Some theoretical and numerical work has be been done in this regard [BW4], and is still in progress.

ii) "Fully Galerkin" approximations. Most numerical optimization schemes are iterative in nature and require a function evaluation on each iteration. Thus when they are used for PDE-based inverse problems it is necessary to (approximately) solve a PDE at each iteration, which can be quite expensive. If these PDE are themselves solved via iterative methods, then the PDE solution for one iteration of the optimization scheme can provides a good "initial guess" for next iteration. This motivates the study of iterative PDE solvers for use in inverse problems. The "fully Galerkin" approach to approximating the solution of time-dependent PDE is simply to discretize the space and time variables by the Galerkin method, obtaining a large sparse linear system which is to be solved in "one shot" rather than by time-marching. A PDE solver based on this approach lends itself naturally to the use of iterative methods.

In [Wa], the theoretical convergence properties of fully-Galerkin methods applied to abstract parabolic PDE in inverse problems are established. Currently in progress is work to numerically investigate the practicality of these methods with iterative solvers in optimization schemes.

iii) The Costate Method. In iterative optimization schemes, typically not only must the function be evaluated on each iteration but the *gradient* must be also. If the unknown parameter has, say, m degrees of freedom, then most methods of computing the gradient require m additional PDE solutions, which can be extremely expensive. An attractive alternative is the costate approach which requires only one additional PDE solution instead of m.

This suggests that the costate method holds potentially enormous savings in computational effort. Unfortunately, numerical experience with this approach in parameter estimation has been puzzlingly disappointing (some explanations can be found in [B]).

In [VW2], the costate method for inverse problems involving linear evolution equations is presented. In addition to some useful positive results concerning convergence, some striking counter-examples are provided and carefully analyzed, both theoretically and numerically. It is hoped that the advantages as well as the subtleties of the costate method will become more fully and widely understood.

iv) Multilevel Methods. For inverse problems in which a distributed parameter is being sought, investigation has been made into the use of multilevel methods. This has been motivated in part by the remarkable success of multigrid methods for certain PDE. The model problem under study here was the "electrical impedance tomography" (EIT) problem of determining interior conductivity from boundary data. One formulation of the problem is based on energy minimization and yields a coupled system of nonlinear elliptic PDE for the inverse problem. In [BMW], an attempt was made to use "standard" multigrid methods (i.e., based on elliptic PDE) to solve the EIT problem thus formulated. However, the nonlinearity of the system gave rise to a certain non-coercivity which caused the multigrid methods to fail.

Another approach to using multilevel methods in inverse problems is presented in [MW]. In that work, a regularized least-squares approach to the EIT problem was studied. Since the presence of the regularization term made the problem essentially elliptic, the question was again investigated as to whether standard multigrid approaches could be used. These efforts met with some measure of success.

v) Block Power method for ill-posed problems. Inverse problems involving distributed parameters can be severely ill-posed, with the Hessian of the least-squares functional having zero as an accumulation point of the spectrum. For problems where this in known to be the case, a "block power" method is presented in [VW1]. This is an iterative method which provides a partial eigendecomposition of the Hessian. It has the advantage of rapidly obtaining that part of the spectrum which is "preferred" by standard regularization techniques (e.g., Tikhonov), and it is lends itself very naturally to a parallel implementation. Work is currently underway to demonstrate the practicality of this method on a large-scale inverse problem.

vi) Frequency domain techniques. Traditional least-squares techniques for PDE governed problems involve a cost or penalty criterion in the time domain. For many problems involving multiple frequency or node excitation in the data, these techniques are extremely computationally intensive. In [VW], [BYW], [BYW2] the authors developed frequency-domain based techniques that involve least-squares criteria for both frequencies and amplitudes of excitation. Both theoretical and computational results for these techniques were given. In [BWI], these techniques are shown to be far superior to least-squares time domain criteria for PDE vibration problems. Development of the frequency domain based criteria represent a major contribution to least squares inverse problem techniques and have become the standard for many computational problems.

VII. Boundary Control Problems (Banks, Ito, King, C. Wang, Y. Wang)

The linear quadratic regulator problem for the systems governed by PDE, such as heat transfer, wave propagation and vibration of flexible structure, were considered in [BI1],[It1] when controls are applied pointwise (as in the piezoceramic actuators discussed above) or through the boundary. Such control problems are formulated as linear control systems with unbounded input operator. A major effort, was undertaken to develop a mathematical framework based on a variational formulation that treats a wide class of practical applications and provides a systematic procedure for developing approximation methods by finite dimensional systems [BIW1], [BIW2]. Theoretical and computational aspects of the optimal feedback synthesis based on the solutions to operational Riccati equations were investigated. Research was completed concerning modeling and control of a multiple component structure composed of flexible and rigid bodies (see [K]). Issues of wellposedness, stabilizability and computation of optimal controls using convergent finite element approximation schemes and linear quadratic regulator theory based on the theory in [BI1], [BIW1], [BIW2] were investigated. The use of piezoceramic patch pair actuators for control of this structure was also considered. Observations were made regarding the effect of the patches on deceleration of gain convergence. The LQR problem for the Euler-Bernoulli beam with Kelvin-Voigt damping and piezoceramic patch pair actuators is discussed in [BIKi]. This problem was also part of an investigation into the use of the linear controller in certain nonlinear beam models. Results include a series of examples regarding the effect of patch placement and size on the control of various initial conditions. Investigations are ongoing into characterizing the continuity (or lack thereof) of the functional gains.

In [BIW3], motivated by applications in noise suppression as discussed above, wave equations with damping in the boundary conditions were considered. A method (based on energy multiplier) to assure the uniform preservation-under-approximation of exponential stability is obtained and several numerical schemes are shown to have such a property using our method. In particular, it was established that a polynomial based Galerkin approximation scheme and a mixed finite element type of approximation scheme can preserve the natural decay rate in the infinite dimensional model equation. It is also demonstrated by numerical computations that the most commonly used approximation schemes such as finite element and finite difference schemes do not have this property. In a continuing effort, we have established in [PW1] precise estimates for the spectrum of the finite dimensional approximation by finite difference and the finite element methods of the above wave equation. It clearly indicated that these two approximation schemes introduce extra spectral points that tend toward imaginary axis as the index of the approximation increases.

Although the investigations in the efforts described above provides valuable information for the selection of the approximation schemes, it does not answer a key question in the approximation of the infinite dimensional control problem which is the uniform stabilizability of the approximate control problems. In [PW2], the authors investigated the relationship between the uniform stabilizability and the asymptotic behavior of the stabilizability margins for the approximate control systems. It was found that a sufficient condition for the uniform stabilizability of the approximate control systems which satisfy the uniform equivalence of the operator norm condition is that the stabilizability margins of the finite dimensional control systems stay uniformly away from zero.

Motivated by the unbounded control input problems arising in the piezoceramic controllers described above, C. Wang considered the pointwise control problem: find a control

function $u_{opt} \in L^2(0,\infty;\mathcal{R}^m)$ which minimizes the functional

$$J(y) = \int_0^\infty (\|y(t)\|^2_{L^2(\Omega)} + \|u(t)\|^2_{\mathcal{R}^m},$$

where y satisfies the wave equation

$$\begin{aligned}\partial_t^2 y(t,x) &= \Delta y(t,x) + \delta_{x_0} u(t), \quad t>0, x\in\Omega \\ \partial_\eta y(t,x) &= -\alpha\partial_t y(t,x) - \beta y(t,x), \quad t>0, x\in\partial\Omega.\end{aligned}$$

A similar problem over a finite time interval with Dirichlet boundary condition was considered by J.L. Lions and Mayeres. While these authors used special techniques to establish the well-posedness of the control problems, no clear definition of the functional spaces in which the actual control system was given which makes it very difficult to consider approximation of a such system. In [W2] the author established the well-posedness of the control problem described above and the space in which the control system is defined is clearly specified and useful characterization of the physical spaces are given.

Several nonlinear models for beams were developed that more accurately describe two types of complex dynamics exhibited by flexible composite materials. The first describes large deformations exhibited by beams constructed of flexible composite materials; well-posedness and control issues for this model are still under investigation.

The second nonlinear model describes the nonlinear stress-strain behavior exhibited even in the small deformation range by flexible composite beams as a result of the linear elastic–nonlinear epoxy matrix interaction. This type of behavior is widely discussed in the composite material literature (see [BKi] for specific references), although little agreement as to the form of the constitutive relation for stress–strain is found. To describe this behavior, we consider a general model of the form $\rho y_{tt} + M_{xx} = g$ where ρ, $y(t,x)$, and $M(t,x)$ are linear mass density, transverse displacement, and internal moment, respectively and $g(t,x)$ is some external forcing function or control. If we assume that stress is a nonlinear function of strain and a linear function of strain rate and make small angle assumptions, then we can write the moment as $M = f(y_{xx}) + c_D I y_{txx}$, where $c_D I$ is the damping coefficient and f is a nonlinear function of strain (y_{xx}). We consider f to be of such a form so that the special case of the Euler-Bernoulli model is obtained upon linearization. Investigations into the wellposedness of this model are underway.

Control of this model was addressed by utilizing smart structure technology, specifically, by using piezoceramic patch pair actuators to induce pure bending moments. A control scheme was developed by computing the solution to the linear quadratic regulator problem for the corresponding linearized system and applying the controller to the original nonlinear system. Preliminary results show for the nonlinearity $f(x) = arctan(x)$ that the linear controller behaves the same way in the nonlinear system as in the linear system. The uncontrolled nonlinear dynamics show more damping than the uncontrolled linear dynamics. The model and the control results can be found in [BKi].

VIII. Numerical Approximations of Delay and Integro differential equations (Ito, Kappel)

An effort was undertaken to develop accurate, robust and implementable numerical schemes for solving a general class of functional differential equations and to analyze convergence properties of the proposed schemes using the Trotter-Kato approximation theorem

for strongly continuous semigroup. In [IK7] the authors developed a general framework for the approximation of delay differential equations that is associated with the Padé approximation of the exponential function. Using the general framework two families of concrete approximations based on piecewise polynomials were constructed. The families include the existing numerical methods for delay differential equations as special cases and for the two families we also prove convergence of the adjoint semigroups and uniform exponential stability which are essential properties for approximation in linear quadratic control problems involving delay differential systems. In [ITM] a fully discrete numerical scheme based on Lanczos-tau methods with piecewise Legendre polynomials was developed for delay differential equations. The scheme has infinite order of accuracy both in time and the delayed argument and can be implemented in a robust and efficient manner. Numerical results illustrating the behavior of the method when faced with difficult problems were obtained and a comparison study with the existing methods was made. In [IT] approximation methods for singular integro-differential equations of neutral type were developed. The study is based on a well-posed state-space formulation in [BI2] and uses a semigroup theoretical framework to analyze the convergence of semi- and fully discrete numerical schemes. Feasibility of the fully discrete scheme is demonstrated by applying it to Able-Volterra equations of the first kind and a singular neutral functional differential equation. In [IKP] an efficient algorithm for computing solutions to a class of models for size-structured populations was developed. The algorithm is based on the method of characteristics and rate estimates for convergence was established and numerical examples demonstrating the behavior and efficiency of the scheme were given.

IX. Variational and Semigroup Formulation (*Ito*, *Kappel*, *Miller*)

Both variational and semigroup theoretic methods were applied to analyze solutions to partial and functional differential equations. In [BI2],[IK9],[It2] the authors developed a state space formulation on the weighted L^p spaces and the space of continuous functions for a class of integro-differential equations arising in aeroelastic dynamics. A linear semigroup theory was applied to show well-posedness, stability, and regularity of the corresponding solution. In [IKS],[IK8],[IK10] an approximation framework for solutions to linear and semilinear Cauchy problems in Banach spaces was developed using a variational formulation and the Trotter-Kato theorem for strongly continuous semigroups. The framework is applied to analyze convergence of numerical methods for the state-dependent delay differential equations, parabolic equations, abstract wave equations and the Navier-Stokes equations. Linear Cauchy problems in Banach space for a system whose generator depends on a parameter was considered in [It3]. Differentiability and the sensitivity equation of the solution with respect to the parameter were discussed. Solutions to the sensitivity equation provide a direct calculation of the gradient of the solution which can be used in both optimal design and parameter estimation problems.

Miller (in joint efforts with Burns and Liu) developed an abstract well-posedness framework using semigroup techniques. This framework was applied to a general class of partial functional differential equations arising in the modeling of viscoelastic and thermoviscoelastic systems. This work also included the proof of convergence of an approximation scheme developed by R. H. Fabiano and K. Ito based on a non-uniform partitioning of the history interval as well as extensive numerical studies. The details of this work can be found in [BLM1] and [BLM2].

X. Homogenization Models for Structures (Banks, Miller)

In early efforts on modeling of truss and lattice structures [BR1], [BR2] it was discovered that traditional structural models provide adequate description of low frequency vibrations for such structures. However, for higher, multiple frequency vibrations, a different modeling approach is needed. Banks and Miller (in collaboration with Cioranescu and Aloh Das - experimental efforts at Phillips Lab - Edwards AFB) undertook a research effort on application of homogenization techniques to problems on lattice structures. Initial efforts with models for an experimental 2-D at Edwards AFB were most encouraging. The goal of subsequent research has been to derive a "simple" equation on a simply connected domain which accurately approximates the behavior of a given system on a domain Ω_ϵ with periodically distributed holes. The idea is to show that the solution of the original problem is bounded independently of ϵ where ϵ is a small parameter denoting the period of the structure. One then extends the solution to all of Ω (which is Ω_ϵ with the holes "filled in") and obtains the "homogenized" equation by letting $\epsilon \to 0$. Complete details of the derivation of the homogenized model for a second order problem on a two-dimensional grid are given in [BCM]. While this paper is introductory and expository in nature, a new result to be found herein is the derivation of the homogenized model for a time-dependent problem with Kelvin-Voigt damping and general (nonzero) initial conditions. Efforts toward generalization of the homogenization arguments thus far have led to the construction of extension operators which take functions defined on Ω_ϵ to all of Ω where $\Omega_\epsilon \subset R^n$. These extension operators preserve bounds on the derivatives of arbitrary order ℓ. This work is detailed in [M2].

In applications of homogenization techniques to specific physical problems, Miller also developed a code to analyze the homogenized model derived by H.T. Banks, D. Cioranescu and D.A. Rebnord for the vibrations of a two-dimensional grid. The frequencies predicted by the model compare favorably with those observed in experiments with an actual grid performed by Banks and A. Das at the Air Force Astronautics Lab (now part of the Phillips Lab) at Edwards AFB. This work is discussed in [BCDMR]. Miller also studied the eigenvalue problem for a class of periodic elastic structures with one dimension large compared to the other two. This class includes trusses and towers. In this work, methods of asymptotic analysis were used to transform a three-dimensional elastic system to a pair of uncoupled "beam-like" (fourth order) equations describing the transverse motion of the structure and a second order equation describing the longitudinal motion. These are equations of one variable (the position along the length) having periodic coefficients; hence, they lend themselves to the homogenization techniques discussed in the previous paragraph. Numerical studies of the homogenized equations and the equations resulting from the first limit process revealed a very good agreement between the two. The error introduced by the first limit process, however, is such that a derivation of higher order "correctors" is necessary. The derivation of the homogenized model is in [M1], and a discussion of the numerical results will appear in [M3]. The work of deriving the correctors is still in its early stages.

XI. Time Discretization for Infinite Dimensional Control Systems (C. Wang)

When a control law is implemented using a computer controlled device, the observation and control action are digitized functions of time. In [RW1] and [RW2], the authors have considered issues arising from the discretization of infinite dimensional control systems. While one is able to show that under reasonable conditions, most of the discrete control systems behave similarly to its continuous time version, the results also indicate that the discretiza-

tion may introduce serious problems if the system model equation is an undamped or lightly damped hyperbolic partial differential equation. In fact, the authors give examples of systems that can be controlled reasonably in the continuous time case, however, the discretized versions of these systems are unstabilizable for arbitrarily small sampling intervals.

Multigrid techniques have been successive in speed-up the solution of some elliptic partial differential equations. The efforts in [RW3] are an attempt to use the basic ideas behind the multigrid method to improve the speed of solving control problems. In particular, it is shown on a model control problem that involves a parabolic control system that the slow-down of the convergence rate as a function of the approximation index can be eliminated by using a multi-level algorithm in the solution of the Lyapunev matrix equation. This idea is then generalized in the design of a multi-level schemes for solving the Lyapunev equation and the Riccati equation. The numerical experiments in this investigation indicate that substantial saving in computational time is obtained by this algorithm.

XII. Modeling and Computation in Groundwater Analysis (Banks, K. Black)

K. Black, under the direction of Banks and Fitzpatrick has focused on flow and the transport of contaminants within a porous media system. The problems are part of efforts on environmental restoration recently begun by the group.

Because there are existing contaminants within groundwater systems a crucial issue in groundwater modelling is the estimation of the various parameters of a given system. There are two opposing views as to how this should be accomplished. The first is that the parameters should be measured directly, and the second is that the parameters should be calculated using the information found from the groundwater systems. The advantage of the first viewpoint is that it is a straight-forward process to accomplish the measurements. The problem, however, is that it can be quite expensive to do this and there is no formal standards to determine how many measurements would be required for a given groundwater system. The advantage of the second view is that it is not as expensive. The drawback is that there is little confidence in the many numerical approximations that have been developed. Again, there is no consensus on how to determine a good match with the parameters calculated and the actual parameters.

Due to the various difficulties found in groundwater modelling several trips were made to Tyndell AFB, FL. The purpose of the trips were twofold. First, it was an opportunity to interact with scientists who have experience in groundwater modelling and who can demonstrate some of the real problems that have to be solved. The trip was also made with a view to developing long term ties to people within the field.

Motivated by these interactions and problems, Kelly's principle research activities within the past year have been the development of groundwater models. In order to do this an initial survey of literature was performed, and contacts with people in the field were initiated. The results of these activities were used in the development of computational models. One of the more difficult problems in groundwater modelling is how to compensate for inhomogeneities within the groundwater system. The parameters describing the media are subject to variations. The approach that has been adopted is to divide the computational domain into many subdomains.

The multidomain approach is useful in that it overcomes certain limitations. A significant problem is how to approximate the parameters of a groundwater system so that it fits data obtained from field experiments. Without dividing an aquifer into zones the parameters

cannot be determined uniquely. The approach adopted is to divide the system into zones and assume that the parameters are constant within each zone but are allowed to vary between zones.

Dividing the computational domain has another important advantage. By dividing up the domain some of the computational issues can be addressed. The approximation of porous media flow has two crucial issues associated with it. First, the computational domain itself is very large. The physical problem may be spread out over an area that is measured in the thousands of square meters. Second, the time span that is of interest is also large. In some cases the time period of interest is measured in tens to hundreds of years. By dividing up the domain into smaller pieces the computational burden can be made more tractable.

Once the domain is divided into smaller pieces, the computational burden can be more easily divided using a distributed computing system. The goal is to implement the approximations on a massively parallel computing system with the greatest efficiency. The computational models examined have been designed for implementation on a distributed environment. The principle thrust of this research is to decouple the subdomains to the greatest extent possible. Once this is done the computations can be carried out with a minimum amount of communication.

The algorithms studied so far implement a multi-domain Chebyshev collocation technique. The enforcement of the subdomain interface conditions has been carried out using a reduced polynomial method. Rather than use a polynomial of full degree to approximate the interface a polynomial of one less degree is used. The result is that the subdomains are decoupled except for adjacent subdomains. The calculations can be carried out using only information from adjacent subdomains rather than using information from every subdomain.

This new algorithm results in an efficient implementation. Implementations of the algorithm include a 2D code that has been used on the Intel iPSC/860 at ICASE, NASA Langley, VA. The code allows for up to 32 subdomains and allows for anisotropic flow. The code is an approximation based on a nonlinear coupled transport equation.

Current work revolves around implementing the algorithm for a full 3D approximation. The goal is to use the reduced polynomial technique along with the penalty method to approximate the equations using an implicit Adams-Bashforth scheme on the interior of the subdomains while employing an explicit Dufort-Frankel scheme at the subdomain interface. Experience on related test examples has been obtained in [Bl1], [Bl2], [Bl3], [Bl4].

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